copies
available

'66

LARGE-SCALE MOTIONS OF THE ATMOSPHERE: CIRCULATION

Edward N. Lorenz

Massachusetts Institute of Technology, Cambridge, Massachusetts

The atmosphere which surrounds our earth is in a state of continual motion. Energy received from the sun ultimately maintains this motion against the dissipative effects of friction. The atmosphere thereby offers a challenging collection of problems to the theoretician. Among these problems, a central one is the following: Given an atmosphere which surrounds a planet and is driven by heat received from a sun, to deduce the resulting global circulation within the atmosphere of the planet, on the basis of the governing physical laws. Other closely related problems are the deduction of specific features of the circulation from the governing laws, and the deduction of certain features of the circulation after taking for granted the existence or nature of certain other features.

On the surface these are straightforward problems in fluid dynamics. In problems of this sort the governing laws are generally expressed as a system of partial differential equations. The motion is inevitably influenced by the fields of pressure and density, and it is logical to regard these fields along with the motion as constituting the circulation.

Our central problem begins to appear more formidable when we express it in more detail. For our own atmosphere, we might state it somewhat like this. "Let there be given an atmosphere, which consists of a specified total mass of a gas of specified chemical composition, together with smaller variable amounts of certain impurities. Let the atmosphere surround an earth of specified mass, radius, and rate of rotation, which serves as both a source and a sink for some of the atmospheric impurities, and whose surface possesses mechanical and thermal properties with specified geographical distributions. Let the earth and its atmosphere receive energy from a sun whose total energy output and energy spectrum are specified. From the physical laws which govern the mechanical and thermodynamic state of the atmosphere and its environment, and the acquisition and removal and internal changes of phase of the impurities, we are to deduce the nature of resulting circulation."

Let us examine this problem for a while. Is it really possible to deduce the properties of the circulation from the laws which govern it? In fact, do the laws even determine the circulation?

There are many facets to these questions. I shall begin by enumerating some of the prominent features of the circulation which have been revealed by observations, and which an acceptable solution to the problem would have to contain.

Possibly the most noteworthy feature of the circulation is hydrostatic equilibrium—an approximate balance between gravity and the vertical pressure-gradient force. This balance demands that the density be proportional to the vertical derivative of pressure. As a result, th pressure and density fields are for practical purposes separate manifestations of the same field—the field of mass.

Nearly as prominent in middle and high latitudes is geostrophic equilibrium—an approximate balance between the Coriolis force and the horizontal pressure-gradient force. This balance, which is the chief feature distinguishing the motion of many rotating fluids from that of nonrotating fluids, demands that the flow be parallel to the isobars on horizontal surfaces, so that, in the northern hemisphere, the winds blow clockwise around high-pressure centers and counterclockwise around low-pressure centers. Further properties of geostrophic motion have been described in detail by Phillips [1963].

The hydrostatic and geostrophic equations may be combined to yield the familiar thermal wind equation, which states that the vertical shear of the wind is directed parallel to the lines of constant density, or, for practical purposes, to the isotherms, on horizontal surfaces.

One feature of the circulation which is very much in evidence is the presence of motions of vastly different scales. Organized circulation systems range all the way from intense high-level vortices covering a major fraction of a hemisphere, through the familiar upper-level waves and migratory cyclones and anticyclones, thunderstorms, and individual cumulus cloud circulations, to the smallest turbulent eddy occurring in the lee of a rock or some other obstacle.

If one takes the pains to analyze the total field of motion and the accompanying field of mass into components of various scales, perhaps by some scheme of harmonic analysis, one finds that the different scales possess different properties. Only the larger-scale motions are approximately geostrophic in their behavior, and the very smallest scales, taken by themselves, do not even satisfy the hydrostatic equation. The total field of motion is nearly geostrophic simply because the major fraction of the kinetic energy of the atmosphere is in the larger scales.

Although the smaller scales of motion sometimes appear rather chaotic, there are certain regularities to be found in the larger scales. One of the most familiar is the occurrence of the trade winds—the easterly winds which are found nearly all the time over the oceans in low latitudes. The famous paper of Hadley [1735] concerning the trade winds marks one of the earliest attempts to account for a major feature of the general circulation. Between the trade-wind belts of the two hemispheres is the intertropical convergence zone, a relatively narrow band in which much of the equatorial storminess is concentrated. Poleward from the trade-wind zones are regions where westerly winds are favored.

At higher levels in middle latitudes are the strong westerly winds, identifiable through the thermal wind equation with the decrease in temperature from low to high latitudes. Some twenty years ago, upper-level observations became plentiful enough to reveal that the westerly winds often culminate in the jet stream, a fairly narrow meandering belt in which the very strongest westerlies are concentrated. The general decrease of temperature from the ground upward, and the existence of a tropopause, above which there is a stratosphere where the temperature no longer decreases with elevation, must certainly be included among the outstanding regularities.

Although some of these large-scale features are virtually always present, it is common experience in most regions of the world that the weather changes from day to day, if not from hour to hour, so that we are not dealing with a steady-state atmosphere. Many of the variations are associated with changes in the positions and intensities of readily identifiable circulation systems, ranging from intense tropical hurricanes and extratropical storms down to minor small-scale disturbances.

If the atmosphere is not a steady-state system, it is just as certainly not a periodically varying system. To be sure, there are periodic components—notably the annual and diurnal periods—just as there is a long-term average state. But although investigators claim to have detected innumerable other periodic components in the observations, I am not aware of any serious claims today that the circulation consists entirely of a superposition of periodic oscillations.

A further significant quantity is the intensity of the circulation. The total atmospheric kinetic energy is roughly equal to the amount of solar energy intercepted by the earth in one hour.

If the statement of the problem and the complex combination of features which must be present in the solution fail to convince us that the problem may be rather difficult, we need only note that the governing system of equations is nonlinear. To many applied mathematicians, no further evidence is needed. The dominating nonlinear terms are those which represent advection—the displacement of features of the field of motion or mass by the motion itself. Since the field of motion is ordinarily not uniform, adjacent portions of any feature will receive unequal displacements, and the feature as a whole will be distorted as well as displaced.

Some of the difficulties may be alleviated by abandoning the exact form of the central problem, and replacing the system of equations governing the true atmosphere by an idealized system of equations. Let us examine some of the ways in which the equations have been modified in various studies of the general circulation.

A common simplification replaces the atmosphere by an ideal gas of uniform composition. Water in its gaseous, liquid, and solid phases is completely disregarded. A parallel simplification replaces the underlying earth by a level surface with uniform mechanical and thermal properties. Oceans and continents with their contrasting heat capacities, and mountains, hills, and smaller irregularities which act as mechanical obstacles, are omitted. Still another idealization regards the incoming solar radiation as a function of latitude only. Differences between summer and winter, and day and night, are eliminated.

There is no question but what the inhomogeneities and asymmetries of the atmosphere and its environment profoundly affect the circulation. Yet the uniform atmosphere moving over a uniform earth has probably received more theoretical attention. Not only is it less complicated, but to many theoreticians it is aesthetically far more pleasing, and, particularly for those whose principal interest is general fluid dynamics, more meaningful.

At this point I wish to digress and mention quite a different type of study. I refer to laboratory models of the atmosphere. The most familiar to the meteorologist are those of Fultz *et al.* [1959] and Hide

[1958]. Here a cylindrical or an annular vessel containing water is rotated about its axis of symmetry, which is vertical, and is heated near its outer radius and cooled near its center or inner radius. The resulting circulation in the liquid is then studied. Although the complete apparatus for the experiment has always been rather elaborate, the actual container in some of the earlier experiments was an ordinary dishpan, and the experiments are probably still best known as the "dishpan experiments."

The circulation in the dishpan may be regarded as another idealization of the atmosphere. Within the limits of experimental control it is the circulation of a uniform fluid over a uniform surface, driven by a heat source which varies only with the distance from the axis. Theoreticians who favor aesthetically pleasing problems but still prefer to study real physical systems have found that the dishpan answers their needs. Outstanding studies are those of Davies [1953, 1956].

The idealizations which we have so far discussed, including the dishpan, all replace the real atmosphere by some physically conceivable system. To be sure, we do not expect to encounter in nature a rotating planet without days and nights, but, in principle at least, we can picture such astronomical monstrosities as a ring-shaped sun with a cold rotating planet at the center. By contrast, many additional common idealizations lead to systems of equations which do not describe realizable physical systems.

One of the most familiar of these is the substitution of the exact hydrostatic equation for the vertical equation of motion. Mathematically this is a major simplification. The new system describes an atmosphere which is incapable of propagating sound waves. Although it is unrealistic because real fluids do propagate sound, it is very practical because sound waves do not seem to interact appreciably with motions of meteorological interest.

A similar idealization is the substitution of a form of the geostrophic equation for the equation expressing the time derivative of horizontal velocity divergence. The new system is incapable of propagating gravity waves [Charney, 1948]. These waves appear not to interact appreciably with motions of meteorological interest, but here the lack of interaction is less certain.

One further idealization is one of the most familiar. The spherical surface of the earth is replaced by a plane, sometimes of limited horizontal extent. The Coriolis parameter is assumed constant, except in those terms where its derivative with respect to latitude appears; this derivative is assigned another constant value, ordinarily denoted by β. The resulting surface is the familiar beta plane, first introduced by Rossby [1939]. Physically one would be hard put to construct a beta plane, yet the beta plane has enjoyed such constant popularity that it seems to have acquired all the status of a physical reality in the meteorological world.

Using some or all of these idealizations, let us see how we might now attack our problem. One straightforward procedure involves expanding each dependent variable in a power series in a parameter ϵ, which characterizes the intensity of the thermal forcing. Since there will be no motion in a dissipative system without forcing, the constant terms in the series for the motion will vanish, and the remaining terms may be found in turn by solving linear equations. It can be shown that the series converge for small values of ϵ, although not necessarily for those values of greatest interest.

It turns out, however, that each term in the series, and hence the solution itself, is invariant with time if the heating is also invariant with time. Small-scale horizontal structure tends to be absent if it is also absent in the underlying surface, although the boundary layer will possess rapid variations in the vertical direction. What we shall have deduced, then, is a steady-state atmosphere devoid of small-scale circulations.

Why does this solution so completely fail to conform with the observations? Have our idealizations created an atmosphere which is incapable of oscillating with time or maintaining a small-scale structure, except perhaps temporarily? The occurrence of nonperiodic motion in the dishpan, which is a similarly idealized atmosphere, indicates that this is not the case. Apparently the solution would be realistic for very weak heating, but for values of ϵ representative of the atmosphere it is unstable with respect to further modes of oscillation, which possess small-scale structures and oscillate nonperiodically. If the steady circulation could be established, and then disturbed slightly, the disturbances would grow until they became dominating features. In the process of growing, the new modes would acquire their energy from the already established modes, and the transfer of energy from one mode to another would be expressed by the nonlinear terms in the governing equations.

We find, then, that the idealizations which we have so far introduced do not overcome the basic difficulties which are associated with the small-scale features and the nonperiodicity of the variations, which in turn result from nonlinearity. Let us try to assess the full effect of nonlinearity upon the problem of deducing the general circulation. If the circulation were steady, we could hope to find this steady circulation analytically. Even if it were periodic, we could hope to solve for a complete cycle. But nonlinearity has rendered the circulation nonperiodic. The equations therefore possess an infinite number of distinct time-dependent solutions, only one of which can duplicate the observed history of the atmosphere, and the probability of selecting this particular solution by chance is zero. If we are to deduce the circulation from the governing laws alone, without using the observed circulation at some special time as a basis for choosing a particular solution, the most that we can hope to do is to duplicate the set of characteristic features of the circulation, which we may call the global climate.

Even this task may not be feasible; there are some systems of equations with the property of intransitivity. For a transitive system, almost all solutions possess the same climate, or set of statistics, but, for an intransitive system, there are two or more climates which a randomly chosen solution has a positive probability of possessing. Whichever climate becomes established will persist forever. There are also real physical systems which are intransitive; under certain conditions the dishpan is such a system.

There is no general rule for determining whether a given system of equations is intransitive, and we must remain in the dark concerning the atmospheric equations. Observations give us no further enlightenment; the atmosphere is a one-shot experiment, and we cannot turn it off and then start it again to see whether another climate develops. Should the atmosphere be intransitive, the most that we can hope to do is to deduce the various different global climates, one of which should conform with observations.

Having decided to find only the statistical properties of the circulation, we might attempt to derive a new set of equations whose dependent variables are the statistics. But the same nonlinearity which has led us to seek only the climate also renders it impossible to obtain a closed system of equations with the statistics as variables. Every effort to complete the system by deriving an additional equation inevitably introduces a new statistic. The only alternative procedure seems to be to return to time-dependent solutions, even though we may not care about them for their own sake, and then compile statistics from them. These statistics will be derived from finite samples, and perhaps not be representative, but this is true also of statistics derived from observations.

We have not finished complaining about nonlinearity. Because the solutions of interest are nonperiodic, we cannot express them in terms of familiar analytic functions. Our one remaining method of solving the equations is by numerical means. To solve partial differential equations numerically, we must first replace the instantaneous field of each dependent variable by a finite set of numbers. Ordinarily these numbers are the values of the variables at a chosen grid of points, although they may be the coefficients occurring in the expansion of the field in a series of orthogonal functions. In essence we have introduced a further idealization.

But now the effect of small-scale features, another product of nonlinearity, comes into play. It is clearly not economically feasible to choose enough grid points or orthogonal functions to give even a remotely adequate picture of the small-scale features. How many cumulus clouds, for example, are in the sky at any one time? We must therefore omit any description of the small-scale features, and deduce the characteristics of only the large-scale motions of the atmosphere.

However, it behooves us not to omit the effects of the small-scale features upon the large-scale motions which we are studying. We may incorporate these effects through the introduction of coefficients of eddy viscosity and eddy conductivity.

It is common experience that we do not possess appropriate numerical values for these coefficients, even to within a factor of 2. The most certain thing we know about them is that they are not constant, so that the use of constant coefficients is a further idealization. Our inadequate knowledge stems from the lack of an adequate theory o. the smaller scales of motion; this in turn is partly due to nonlinearity.

Following these modifications which nonlinearity has demanded, further idealizations may be in order. With small-scale features eliminated, the remaining large-scale circulation generally possesses a far less detailed structure in the vertical direction than in the horizontal. It has therefore proven feasible to replace the entire three-dimensional field of motion by two two-dimensional fields—one in each of two layers. If the geostrophic approximation is also used, a single two-dimensional temperature field may be identified with the difference between the two fields of motion.

Many of these idealizations were originally devised for the purpose of dynamical weather forecasting. Phillips [1956] was the first to apply the methods of dynamical forecasting to the problem of deducing the general circulation. His system of equations contains all of the idealizations which we have enumerated.

Still more recent is the common use of low-order models. Here the number of degrees of freedom, that is, the number of numerical values which must be specified to describe an instantaneous state of the system, is reduced to the point where only the very largest scales of motion remain. These models are so simple that one may actually inspect the few columns of numbers representing a time-dependent solution and get some feeling for what is taking place. They are especially useful for determining just what physical features must be retained in order that specific climatic features may be reproduced [Lorenz, 1962, 1963*b*].

Our latest idealizations all leave us with systems of equations which are readily handled by digital computing machines. In retrospect, it appears that numerical methods afford the only method by which the central problem can be solved, and, even then, it can be solved only when the system of equations is idealized. It is always dangerous to claim that a task is impossible, unless, like the trisection of the angle with the compass and straightedge, it can be proven to be impossible, but the writer's [1964] experience with the very simplest of nonlinear equations, a first-order quadratic difference equation in one variable, strongly indicates that the statistics of nonperiodic solutions cannot in general be found except by numerical means.

Let us recall, then, that dynamic meteorology was a well-developed discipline long before digital computers were dreamed of by most meteorologists. When Richardson [1922] foresaw the power of numerical methods nearly half a century ago, he visualized a team of 64,000 men working together, rather than a computing machine. Even today many of the most capable meteorologists have little access to machines. What, then, were the problems which dynamic meteorologists of earlier generations could hope to attack successfully?

We have already alluded to some of these. First there are problems of deducing specific features of the general circulation. It might appear illogical that we could deduce part of the circulation while neglecting the remainder, when all parts are interrelated. Probably we cannot deduce exact numerical values of any feature without implicitly considering all features, but we can certainly deduce upper and lower bounds.

Take, for example, the problem of explaining why the equatorial regions are warmer than the polar regions, as opposed to the determination of exact temperatures in these regions. Perhaps everybody knows the answer; is it not simply that there is more solar heating in the equatorial regions? The problem may seem too simple to merit further consideration, until we recall that at certain elevations, notably the lower stratosphere, the equatorial regions are ordinarily colder than the polar regions. Perhaps the problem is not so trivial after all.

The problem can be rigorously treated within the framework of a two-level model, such as the one used by Phillips [1956]. Here the variance of temperature serves as available potential energy, the only direct source for kinetic energy. Kinetic energy is continually being destroyed by friction, and must be continually replaced at the expense of the temperature variance. The outgoing radiation, if not regarded as a constant, is taken as an increasing function of temperature, and thereby further serves to decrease the temperature variance. Incoming radiation must then maintain the temperature variance in the face of these other processes, and it can do this only by heating the warmer regions more than the cooler regions. But incoming radiation in this model depends upon latitude alone; it follows that by and large the warmer regions are the low latitudes, and the cooler regions are the high latitudes.

In the real atmosphere the poleward temperature decrease probably occurs for the same basic reasons, but the arguments as we have presented them are no longer rigorous. For example, the incoming radiation, excluding that part reflected back to space, is no longer a function of latitude alone, so that a correlation between temperature and heating no longer demands a correlation between temperature and latitude. To make our arguments rigorous, we probably must incorporate the fact that the effect of the asymmetries does not exceed some critical value. I believe that the argument can be made rigorous for the real atmosphere. I have not seen this done, possibly because the problem has seemed too trivial at first glance.

Similar methods, involving careful consideration of the asymmetries, can perhaps be used to establish the necessity for the existence of other qualitative features of the circulation—possibly the trade winds or the jet stream. I do not know of any such studies where completely rigorous proofs have been offered. Here is a wide open field of research.

There are also problems of deducing certain features when other features are taken for granted. Problems of this sort have perhaps occupied the major efforts of those theoreticians who prefer analytic to numerical procedures. Here alone among the problems we have mentioned is the opportunity to work with linear equations, whose mathematical theory has been so highly developed.

The most familiar examples are problems involving the stability of a steady field of flow. The equations may be idealized to the point of neglecting viscous dissipation and thermal forcing, since it is no longer necessary to explain how the basic flow originated.

Numerous investigators [e.g., Kuo, 1952] have obtained the result that patterns containing six, seven, or eight waves around the circumference of the globe will grow more rapidly than any other modes of oscillation, when superposed upon basic flow patterns resembling those found in the atmosphere. The additional hypothesis [Eady, 1949] that those wave patterns which grow most rapidly when small are those which are favored to remain after reaching finite amplitude affords an explanation for the observed prevalence of waves of this scale.

Somewhat different is a recent study of Saltzman [1963], who assumed statistical properties of the waves and deduced with fair accuracy the observed distribution of the zonally averaged component of the circulation. Perhaps it may be possible to combine this sort of study with the studies of stability, and solve simultaneously for the basic flow and the superposed waves.

Let us now return to the central problem, and observe what success has been attained in deducing the climate compatible with various idealizations. We begin with the simplest model of a forced dissipative system which has been studied, and, so far as I know, the only one whose general nontransient solution has been found analytically [Lorenz, 1962]. It is a low-order model, with 8 degrees of freedom. From our previous remarks, we can already infer than the general solution must be periodic.

Even in this simple model there are upper-level westerly winds, and also cyclones and anticyclones and upper-level waves when the flow is unsteady. Moreover, as in the dishpan experiments, the rate of rotation and the intensity of heating determine whether a steady or an unsteady flow will occur. The model by its very construction is incapable of maintaining trade winds and surface westerlies. Two somewhat less restricted models, one with 14 and one with 28 degrees of freedom, oscillate nonperiodically under suitable conditions [Lorenz, 1963*b*, 1965].

The prototype of general-circulation models, that of Phillips [1956], has 480 degrees of freedom. The features which develop in the low-order models occur here with more realistic structures, and, in addition, trade winds and middle-latitude westerlies occur.

The most recent and least highly idealized general-circulation models are those of Leith [1965], Mintz [1965], and Smagorinsky [1963, 1965]. Each of these models has several thousand degrees of freedom. None of them uses the geostrophic approximation, and all of them have oceans and continents. Leith's atmosphere contains water vapor, while Mintz's earth has mountains. Together, the models show most of the principal features of the large-scale motions, except the prevalence of tropical storms. Smagorinsky's model, with the most detailed vertical structure, exhibits a fairly realistic tropopause.

We have therefore nearly solved the problem of deducing the large-scale motions of the atmosphere from the governing physical principles, within the framework of the idealizations which we have introduced. Some of the principal remaining deficiencies, such as the absence of tropical hurricanes and their influence upon the larger features, can possibly be attributed to faulty parameterization of the small-scale processes, notably cumulus convection. If, a few years from now, we succeed in properly reproducing all of the principal features, we shall, in the opinion of some meteorologists, have attained the Ultima Thule in the study of the general circulation.

There are others who would feel that by merely duplicating the circulation as we have observed it we have done little more than verify the validity of the equations. To appreciate this point of view more fully, let us note that, in a certain sense, the dishpan may be regarded as an analog computing machine, solving its own idealization of the equations governing the atmosphere. To someone who seriously asks, "Why does the atmosphere have trade winds?", the answer, "because the dishpan has trade winds," would have all the earmarks of an elephant joke.

But is the answer, "because the computing machine says so," or even the more informative answer, "because trade winds occur in all the known solutions of systems of equations which closely resemble the atmospheric equations," any more satisfactory? Undoubtedly it is the desire to discover *why* some phenomenon occurs, rather than merely showing that it must occur, which has led many theoreticians to avoid purely numerical problems. As we have noted, there is no shortage of worthy problems of an analytic character available to them.

A little reflection will reveal that there really is much to be learned about the atmosphere from numerical computations, regardless of whether they answer the questions which we consider most fundamental. A particularly welcome feature is the opportunity to perform controlled experiments upon the atmosphere in simulated form, with no risk of rendering the earth unfit for further habitation. We might even learn whether the atmosphere is transitive. I should like to conclude by describing a series of experiments of this sort which is just now bearing fruit.

These experiments are concerned with the extent to which the future state of the atmosphere, or the future weather, can be predicted. Some of the factors involved have been discussed by Thompson [1957], and more recently by the writer [1963*a*]. A mathematical proof that a stated task, such as the trisection of an angle, is impossible constitutes an acceptable solution to a problem, and the possibility of proving that the weather cannot be predicted has appealed to some theoretical meteorologists whose own attempts to forecast the weather have met with less than complete success. We shall not be so much concerned with the philosophical question of determinism, or whether the atmosphere has decided what to do, as with whether it has signaled its intentions to us.

We begin with the now familiar observation that the atmosphere is nonperiodic, except for the annual and diurnal periods and possibly a few minor periodic components. It can be proven that the nonperiodic component of a system varying with periodic and nonperiodic components is completely unpredictable in the far distant future, unless its present state or some past state is known exactly.

In brief, a nonperiodic system is essentially one which never does the same thing twice; any approximate repetition of its previous behavior must be of finite duration. It therefore cannot be predicted to do the same thing twice by any acceptable forecasting scheme. But if there are uncertainties in observing the system, a state which is indistinguishable from some previous state will eventually occur, and no forecasting scheme will have any rational basis for predicting that the system will not do the same thing twice.

There are always appreciable uncertainties in observing the state of the atmosphere, due not so much to faulty observation as to the impossibility of observing the weather everywhere. Even in the United States and Europe, where the observing weather stations are most closely spaced, a system as large as a thunderstorm occurring between stations can go completely unnoticed. Over most of the ocean, except in the principal shipping lanes, there are often gaps in the observations large enough to conceal a fully developed hurricane. It follows that there will be a finite limit to how far into the future we can predict, as long as these conditions remain.

Of course, our observations do not eliminate the possibility that the atmosphere is periodic, with a period longer than the duration of recorded history. However, the established nonperiodicity of some of the simpler idealized systems suggests that this is not the case.

The theory which so neatly asserts the ultimate unpredictability of the atmosphere does not tell us how far into the future we can predict. The rate at which initial errors will amplify may be estimated through numerical experiments.

The first experiment [Lorenz, 1965] was based on the low-order model with 28 degrees of freedom. Here the growth rate of errors varied as greatly as the weather situation itself, but, on the average, errors doubled in about four days.

Slightly more encouraging news has just been communicated by Mintz, following experiments with his system of equations with many thousand degrees of freedom. He finds that on the average about five days are required for the size of the errors to double. The doubling period for the real atmosphere, whatever its length may be, is yet another characteristic feature of the general circulation.

If it really requires as long as five days for typical errors to double, moderately good forecasts as much as two weeks in advance may some day become a reality. If it requires no longer than five days for errors to double, accurate detailed forecasts for a particular day a month or more in advance belong in the realm of science fiction.

References

Charney, J. G., On the scale of atmospheric motions, *Geofys. Publikasjoner Norske Videnskaps-Acad. Oslo, 17*, 2, 1948.

Davies, T. V., The forced flow of a rotating viscous liquid which is heated from below, *Phil. Trans. Roy. Soc. London, A*, 246, 81–112, 1953.

Davies, T. V., The forced flow due to heating of a rotating liquid, *Phil. Trans. Roy. Soc. London, A*, 249, 27–64, 1956.

Eady, E. T., Long waves and cyclone waves, *Tellus, 1*, 35–52, 1949.

Fultz, D., R. R. Long, G. V. Owens, W. Bohan, R. Kaylor, and J. Weil, *Studies of thermal convection in a rotating cylinder with some implications for large-scale atmospheric motions, Meteorol. Monographs*, 1959.

Hadley, G., Concerning the cause of the general trade winds, *Phil. Trans. Roy. Soc., 39*, 58, 1735.

Hide, R., An experimental study of thermal convection in a rotating liquid, *Phil. Trans. Roy. Soc. London, A, 250*, 441–478, 1958.

Kuo, H. L., Three-dimensional disturbances in a baroclinic zonal current, *J. Meteorol., 9*, 260–278, 1952.

Leith, C. E., Lagrangian advection in an atmospheric model, *Proc. Sympos. Long-Range Forecasting*, World Meteorol. Org., Tech. Note 66, 168–176, 1965.

Lorenz, E. N., Simplified dynamic equations applied to the rotating-basin experiments, *J. Atmospheric Sci., 19*, 39–51, 1962.

Lorenz, E. N., The predictability of hydrodynamic flow, *Trans. N. Y. Acad. Sci., 25*, 409–432, 1963*a*.

Lorenz, E. N., The mechanics of vacillation, *J. Atmospheric Sci., 20*, 448–464, 1963*b*.

Lorenz, E. N., The problem of deducing the climate from the governing dynamic equations, *Tellus, 20*, 1–11, 1964.

Lorenz, E. N., A study of the predictability of a 28-variable atmospheric model, *Tellus, 21*, in press, 1965.

Mintz, Y., Very long-term global integration of the primitive equations of atmospheric motion, *Proc. Sympos. Long-Range Forecasting*, World Meteorol. Org., Tech. Note 66, 141–167, 1965.

Phillips, N. A., The general circulation of the atmosphere: a numerical experiment, *Quart. J. Roy. Meteorol. Soc., 82*, 123–164, 1956.

Phillips, N. A., Geostrophic motion, *Rev. Geophys., 1*, 123–176, 1963.

Richardson, L. F., *Weather prediction by numerical process*, Cambridge University Press, 1922.

Rossby, C. G., and collaborators, Relation between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semi-permanent centers of action, *J. Marine Res.*, *2*, 38–55, 1939.

Saltzman, B., A generalized solution for the large-scale, time-average perturbations in the atmosphere, *J. Atmospheric Sci.*, *20*, 226–235, 1963.

Smagorinsky, J., General circulation experiments with the primitive equation. I. The basic experiment, *Monthly Weather Rev.*, *91*, 99–164, 1963.

Smagorinsky, J., Implications of dynamical modeling of the general circulation on long-range forecasting, *Proc. Sympos. Long-Range Forecasting*, World Meteorol. Org., Tech. Note 66, 131–137, 1965.

Thompson, P. D., Uncertainty of initial state as a factor in the predictability of large scale atmospheric flow patterns, *Tellus*, *9*, 275–295, 1957.