How Much Better Can Weather Prediction Become?

Man may have first decided that he could learn to predict the weather after becoming aware that certain regularities mark the sequence of weather events; for example, dark clouds often foretell the outbreak of a heavy shower. No doubt he was subsequently encouraged by discovering that certain other natural phenomena, such as oceanic tides and solar eclipses, could be predicted far in advance with considerable accuracy. Today we are more inclined to base our belief in predictability on the existence of a set of physical laws according to which the present state of the atmosphere and its environment determines the future.

If such laws prevail, it might seem that we have only to perfect the technique of applying them, in order to put weather forecasting on a par with eclipse forecasting. Indeed, such an achievement has been the stated goal of some utopians. Yet recent evidence points against its fulfillment, despite the physical laws. Because of a combination of circumstances, there appear to be certain limitations on predictability which no system of forecasting can ever hope to overcome.

A prerequisite for an accurate forecast of a future state of the atmosphere is an accurate knowledge of the present state or some recent past state. The mere fact that the governing laws picture new states as evolving from earlier states is not sufficient to assure us that this is so. Governing laws also describe new states of the tides as evolving from older states, yet ordinarily we base our predictions of the tides upon the anticipated configuration of the moon, earth, and sun, disregarding the state of the tide at the time we make our prediction.

To be sure, we can say something about future weather from the time of the day and the year alone. We can predict with near certainty that next summer will be warmer than last winter, and, in some climates, drier, in others, wetter. Yet over much of the globe the weather variations of greatest interest are those associated with migratory areas of storm and calm—systems which cannot be foretold by the calendar and the clock alone, and whose progression has not been observed to follow any precisely periodic pattern. No procedure for predicting these variations which does not take the current state of the atmosphere into account has proven itself any better than guesswork.

The weather recognizes no international boundaries; a storm which is centered in France today can appear in Germany or Poland tomorrow. Hence, from its inception a century ago the practice of weather forecasting has demanded a degree of cooperation among even those nations which might have been disinclined to cooperate in other matters. As the various nations have established and continually enlarged their networks of weather stations, they have striven for enough uniformity in their observing procedures to render the data useful to everyone.

With several thousand weather stations reporting winds, pressures, temperatures, humidities, cloud forms, and other weather elements at least twice daily, it might appear that a rather complete picture of an instantaneous state of the atmosphere could be constructed by interpolation. Unfortunately, this is not the case. Even over more populous regions, where neighboring stations are typically a hundred miles apart, an intense thunderstorm between stations may go unreported. Over the oceans and away from the principal shipping lanes and airline routes an entire tropical hurricane may remain undetected. Satellite photographs are now revealing storms and other systems which might not have been discovered otherwise, but they do not capture the atmosphere's complete three-dimensional structure (see Dr. William K. Widger's "Meteorology by Satellite," Technology Review for July/August, 1968, p. 35).

Plans to correct the most serious deficiencies through an international "World Weather Watch" are in progress. Yet no matter how dense the network of stations may become, there will always be still smaller irregularities between stations which
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will be unrecorded. There is considerable justification for the claim that the useful information is contained in a smoothed weather pattern, with the irregularities eliminated, but if an irregularity occurs at an observing station, and is not recognized as such but treated as a point observation of a smooth trend, we shall obtain the wrong smoothed pattern. Thus one of the prerequisites for a perfect forecast—a perfect knowledge of current conditions—cannot truly be attained.

Craft or Mathematics
Traditionally weather forecasting has been a subjective procedure; if not an art, it has at least been a craft rather than a science. The forecaster begins with the present and recent past observations; so that they will not constitute an unmanageable jumble of facts, they are arranged as a set of weather maps. The forecaster first analyzes the most recent maps—he or one of his colleagues will have analyzed the earlier maps before issuing the previous forecast—identifying such systems as high and low pressure areas, warm and cold air masses, and fronts. He then estimates the future position, intensity, and shape of each system, taking care to introduce new systems whose formation seems to be indicated, and to remove systems which appear to be disintegrating. From his prognosticated weather pattern he ultimately deduces the weather conditions at specific points of interest.

At times he may make use of the governing physical laws, but ordinarily he bases his estimate on the way in which the existing systems have been behaving, and on his knowledge of how similar systems have behaved on previous occasions. He must be able to decide when the present weather situation truly resembles some earlier one with which he is familiar, and when the resemblance is only superficial. He must learn to recognize the various signs of storm development and decay, just as the physician learns the symptoms of specific illnesses.

One can well imagine that there will be occasions when the forecaster relies too heavily upon one sign and too little on another. At times the current weather situation will be unlike any which he can recall. A poor forecast will be the inevitable result. Many forecasters regularly conduct post-mortem discussions, and sometimes, after a forecast has failed, they are able to identify some feature which, had it been given greater attention, would have led to a proper forecast. On other occasions they may find no indication that what did happen was about to happen. Yet the governing physical laws seem to imply that the indication must have been there. The resulting disillusionment with current subjective methods has led some forecasters to seek procedures which, once perfected, will no longer rely upon human judgment and alertness.

The most highly developed objective method of forecasting is a dynamical method, popularly known as "numerical weather prediction." Here the governing physical laws are formulated as a system of differential equations. The particular solution of these equations for the case when the initial conditions represent the present state of the atmosphere is then sought. The method was proposed many years ago, but the equations are so highly nonlinear (effects disproportional to causes) that the only known methods of solving them are numerical (a brute-force arithmetical procedure) and these were impractical before the advent of high-speed digital computers.

In the United States the method became operational in the middle 1950's; refinements are continually being added. Numerical forecasts prepared by a central computer at the National Meteorological Center are issued to the various forecasting offices. Such forecasts are ordinarily presented as sets of prognostic weather maps, indicating the expected locations and intensities of the various weather systems. Under current procedure, the local forecaster is not bound by the numerical forecast if his judgment tells him that something else should happen; however, the numerical forecast is there as an additional piece of information, and he is likely to be heavily influenced by it.
In principle this dynamical method should produce an optimum forecast, but in reality there are several reasons why the forecasts fall short of perfection. First of all, the governing laws are not strictly deterministic. We need not invoke Helsenberg’s Principle of Uncertainty to justify such a statement. It is sufficient to note that the weather is affected at least to some extent by human activity, which we hesitate to consider predetermined. Local cumulus-cloud convection, for example, may be initiated or intensified by fires. Such inadvertent weather modification has in recent years been supplemented by conscious attempts to alter the weather for man’s benefit (see, for example, Dr. Frederick Sargent’s “Weather Modification and the Biosphere,” Technology Review for March, 1969, p. 42).

A more important consideration at present is our incomplete knowledge of the governing laws. We do not know, for example, precisely what determines when a cloud consisting entirely of minute water droplets will become converted into a cloud containing larger drops, which will then fall out as rain. Such lack of knowledge can disrupt a forecast far more than any uncertainty as to the location of fires and other man-made features.

However, the current failures of numerical forecasting stem most of all from our inability to formulate the laws as equations which can be solved by digital computers, without distorting the laws in the process. The familiar partial differential equations treat the atmosphere as a continuum, but the computer is a finite instrument, and it must represent the state of the atmosphere by a finite collection of numbers. Usually the numbers are values of the weather elements at a prechosen network of points, and finite differences from point to point and moment to moment replace the partial derivatives (smooth gradients) of the equations. Inevitably, some of the finer details such as thunderstorms are omitted, not only at the initial moment (when they are likely to be unobserved in any case) but throughout the period of the forecast. Yet these details exert a continual influence upon the larger systems, and cannot be disregarded. We generally try to include their effects by introducing coefficients (or turbulent viscosity and turbulent conductivity (for example, by acknowledging the existence of smaller-scale phenomena, without being specific about them), but we do not know the most appropriate values for these coefficients, nor have we proven that appropriate values even exist. Our inability to observe the present state of the atmosphere without error is therefore accompanied by a similar inability to extrapolate the state into the future without error, if dynamical procedures are used.

Prediction Without the Laws
A forecasting procedure does not have to be dynamical to be objective. In recent years considerable attention has been devoted to empirical methods; these have also depended heavily upon the computer.

Prediction by linear regression is the empirical method whose mathematical theory has been most highly developed. Here we express the predictive value of some observable weather element as a linear combination of a chosen set of observable predictors; the coefficients of this combination are discovered empirically. No maps need be analyzed; and the specific weather conditions do not have to be inferred from prognostic maps.

For special tasks where methods currently in use are significantly but only slightly better than guess work, such as predicting the general trend of the weather a month in advance, linear regression may give the best results. For the regular daily forecasts the method has not compared favorably with subjective or dynamical procedures. Evidently the dominating terms in the governing equations are too highly nonlinear to be readily approximated by linear functions of the present and past weather.

We therefore turn to an empirical method which incorporates all the inherent nonlinearity—the method of analogues. Here the computer examines the entire recorded history of the atmosphere, or
Instability may be illustrated by a simple equation, as below. Starting with an initial number \( X_0 = 0.84 \), we have generated a sequence of numbers \( X_1, X_2, \ldots \) using the difference equation \( X_{n+1} = 1.64 - X_n^2 \) (i.e., \( X_1 = 1.64 - (0.84)^2 = 0.9344 \), etc.). The heavy line connects successive numbers of the sequence. The dots show the numbers which would have been predicted by the same equation if \( X_0 \) had mistakenly been observed as \( \frac{1}{2} \). The initial error of 0.02 would have increased after five steps to 0.176 and after 10 steps to 1.718, whence the prediction 10 steps ahead would have been worthless. The equations governing the atmosphere are vastly more complicated than simple difference equations, but mathematically the phenomenon of instability is similar.

In practice the method has not been particularly successful. For predicting one day in advance, it might be sufficient to have the analogue state resemble the current state over a rather limited area; for predicting several days ahead the resemblance should cover a fair portion of the globe. Reasonably complete three-dimensional states of the atmosphere have been observed on a daily basis over the northern hemisphere for no more than 25 years. The chances of finding a good analogue for a given state within this period are extremely small. To be competitive with dynamical forecasting as it is currently practiced, the analogue method would probably require many thousands of years of recorded weather data.

In principle the analogue method, like the dynamical method, should yield an optimum forecast. If two states of the atmosphere are alike within the limits of observational error, either the subsequent states will be alike, and the analogue method will produce the correct forecast, or the subsequent states will not be alike, in which case no systematic procedure would have produced the correct forecast on both occasions.
The upper curve shows an idealized spectrum of atmospheric kinetic energy against wavelength. The area under the curve between any two vertical lines is proportional to the amount of energy contained in systems having wavelengths between the values indicated. The curves labeled 15 minutes, 1 hour, 5 hours, 1 day and 5 days are theoretically determined spectra of the mean-square error in predicting the velocity field at those time ranges. Thus, scales of motion greater than 2500 kilometers are almost perfectly predictable one day ahead, while scales less than 625 kilometers are almost completely unpredictable at that range. (Because of numerous assumptions entering the computations these results should not be regarded as the final word.)

These considerations indicate that perfect weather forecasting is at present unattainable, but they do not by themselves preclude the possibility of eventually producing forecasts of high quality at both short and long range. Although we cannot wait long enough to acquire the data needed to make the analogue method operationally feasible, there is no obvious upper bound to the accuracy with which the weather may some day be observed, nor, aside from the slight lack of determinism, to the precision with which the laws may be formulated.

**The Growth of Small Differences**

The additional circumstance which places a limit upon the ultimate accuracy of weather prediction is the atmosphere’s *Instability*. Specifically, two states of the atmosphere which closely resemble one another will, in evolving according to the governing laws, ultimately develop into vastly dissimilar states. Stated otherwise, two solutions of the governing equations, originating from slightly different initial conditions, will ultimately diverge (page 42).

How can we be certain that this is so? Mathematical theory has not advanced to the point where we can examine any given system of nonlinear equations and say whether the general solution will be unstable. Our principal evidence is the nonperiodic nature of the atmosphere, which we have already mentioned.
If a system is stable, it will in the absence of non-periodic external influences acquire a completely periodic behavior. Stability and periodicity must be carefully defined to render such a statement capable of rigorous mathematical proof, but the general line of reasoning may be presented qualitatively.

The number of possible states of the atmosphere, each bearing no resemblance to any other, is limited. Hence, if the atmosphere is observed over a sufficiently long interval, a pair of reasonably good analogues can be found; the longer the interval, the better the analogues. If the atmosphere were stable, it would behave similarly following the occurrences of either analogue state. History would repeat itself, and the atmosphere would be periodic. All our observations clearly indicate that this is not the case. (Of course, there is always the possibility that the atmosphere really is periodic, with a period longer than its observed history, but this is highly improbable.) We may therefore take it that the atmosphere is not stable.

Consider now two states of the atmosphere, one of which is the exact present state, and the other of which is the best attainable estimate of the present state, containing the inevitable errors of interpolation. These states, we have seen, will eventually evolve into states bearing little resemblance to one another. It follows that even the most perfect prediction technique cannot yield good forecasts at indefinitely long range. Imperfections in the technique will only aggravate the problem.

It is the instability of the atmosphere which makes it less predictable than tides and eclipses. It is instability which renders empirical methods of prediction only moderately successful.

Knowing that we cannot predict into the indefinite future, we face the question, "How accurately can we hope some day to predict the weather at any specified range?" The answer to this question depends upon how rapidly separate solutions of the atmospheric equations diverge from one another.

It is convenient to regard the difference between any two states of the atmosphere as an "error"—the error one would make if he mistook one state for the other. We then face the question, "How rapidly do small errors grow?"

This was one of the questions asked in the early 1960's by a Panel on International Meteorological Cooperation headed by J. G. Charney, charged with evaluating the probable effectiveness of an all-out effort to improve the world-wide observation system. The Panel noted the possibility of a dynamical approach to the error-growth question; separate solutions, initially slightly different, of the equations which had proven effective in numerical weather prediction could be determined and compared.

We have seen that the equations of numerical forecasting are not exact; neither are they the product of a single person or a single working group. Thus it was inevitable that different investigators would develop different systems of equations, each with its own distinctive features. By the early 1960's three groups—those of J. Smagorinsky at the U.S. Weather Bureau, Y. Mintz at U.C.L.A., and C. E. Leith at the Lawrence Radiation Laboratory—had developed equations which seemed suitable for investigating the growth rate of errors. In each case the state of the atmosphere was described by a few thousand numbers.

Following a special conference in 1964, each investigator agreed to use his equations for this purpose. The results of the separate computations did not agree. Mintz found that after an initial period of adjustment, small errors in winds and temperatures would tend to double in about five days. Smagorinsky deduced a considerably slower growth rate, while Leith obtained no systematic growth at all. It appeared, however, that Leith's atmosphere was varying nearly periodically, whence—by the above "stability" reasoning—little error-growth was to be expected. In Smagorinsky's and Mintz's experiments, the growth rate subsided as the errors became larger.
In their report to the National Academy of Sciences, the Panel concluded that a reasonable estimate of the doubling time for small errors was five days. It was felt that the hoped-for improvement in observation might reduce the initial error of observation to one-eighth of the tolerable error of prediction. Thus day-to-day forecasting up to two weeks in advance (i.e. three doubling times) appeared possible, and was accepted by some as a goal.

Subsequent studies where the state of the atmosphere was represented by as many as 100,000 numbers seemed to confirm a doubling time of somewhat less than a week. However, even the most detailed equations cannot circumvent the problems raised by the presence of small-scale features. Thus we can never be sure that the results deduced from the equations are valid for the atmosphere itself. It therefore behooves us to seek other means of estimating the growth rate.

Error-Growth from History
Such means are afforded by an empirical approach, which is based upon the analogue method of forecasting. If two states qualify as analogues, either state is equivalent to the other plus a small error, and the growth of the error may be studied by observing the behavior of the atmosphere following the two states.

In practice we cannot expect to encounter any good analogues within the brief recorded history of the atmosphere. We may therefore observe the growth only of moderately large errors. These errors should have a longer doubling time than small errors (in the extreme, once an error has become as large as the difference between randomly chosen states, it should undergo no further systematic growth). By studying mediocre analogues, we may hope at least to obtain a maximum estimate for the doubling time of small errors.

We have recently completed such a study in the Statistical Forecasting Project of the Meteorology Department at M.I.T. Our basic data have been about 10,000,000 numbers—the elevations of three constant-pressure surfaces, twice daily for five years, at a network of 1000 points covering most of the northern hemisphere. We have compared each state of the atmosphere with each other state occurring within one month of the same time of year, but in separate years, thereby comparing altogether about 400,000 pairs of states.

There are indeed no truly good analogues. In fact, the smallest differences encountered are already more than half as great as the difference between two states chosen at random (which can never double at all). On the average, the smallest errors amplify by nearly 10 per cent in one day; thus it may be inferred that truly small errors would need not more than eight days to double—a result which, incidentally, is in agreement with the numerical experiments.

Presumably, however, the doubling time for truly small errors is considerably less than that of the smallest errors encountered in this analogue study. If we postulate that the eventual cessation of growth, as the errors become larger, is due to processes represented by quadratic terms in the dynamical equations, we can extrapolate our results. We then find that very small errors should double in about 2.5 days.

In both the dynamical and the empirical procedure the state of the atmosphere is represented or described by numerical values of the weather elements at points separated by several hundred miles. The errors which are found to double in several days are therefore exclusively the errors in representing the larger-scale features of the atmosphere. It seems likely that errors in smaller-scale features will double much more quickly. An error in estimating the intensity of a thunderstorm, for example, should amplify at least as rapidly as the thunderstorm itself, doubling in perhaps 20 minutes. At the same time, this error may be instrumental in producing errors in the larger scales.

The Statistics of Errors
A third approach to the question explicitly takes this possibility into account. The new approach is partly dynamical and partly empirical. From the original atmospheric equations, we may derive a new set of equations governing the statistical properties of the errors. The coefficients in the new equations are based upon observed statistical properties of the atmosphere itself—the spectrum of amounts of motion on different scales (or wave lengths) (see chart, page 43).

The Statistical Forecasting Project has also completed a study of this sort. We have derived a system of 20 equations in 20 unknowns; each unkno
technology and makes ever more precise computations. "... It might be supposed," writes the author, "that we could continually improve our predictions by refining our observations. It appears, however, that as long as the larger scales are observed with reasonable accuracy, the advantages to be gained by improvements in observing the atmosphere are slight. ... If we could observe all scales down to thunderstorm scale, a further doubling of precision would gain us only a few minutes." (Photos: Benjamin Lifson)
represents the contribution of one “scale of motion” to the mean-square error. Each scale covers an octave of the spectrum, so that wavelengths from 40,000 kilometers down to 40 meters are included. The study represents a first attempt, and in place of actual atmospheric equations we have used equations for two-dimensional incompressible flow.

We find that when the initial error is confined to the smallest scale of motion, it grows very rapidly, at the same time inducing errors in slightly larger scales. These in turn grow slightly less rapidly, and induce errors in still larger scales. In the course of half an hour, errors in the thunderstorm-sized scales have become appreciable, while after two days the errors have invaded the scales associated with migratory storms. Large errors in all scales are present after two weeks.

If the small initial errors are instead contained in the medium or larger scales, they quickly induce errors in the smallest scales, which then proceed to behave as if they had been present from the beginning. Thus in either event the errors in the most rapidly amplifying scales, i.e., the smallest, will soon dominate the field, and only somewhat later will they succeed in inducing additional errors in the larger scales. In other words, the errors which prevail after a few hours or a few days in any scale will be mainly the result of initial errors in the smallest scales. Now, it might be supposed that we could continually improve our predictions by continually refining our observations. It appears, however, that as long as the larger scales are observed with reasonable accuracy, the advantages to be gained by improvements in observing the atmosphere are slight. For suppose that we somehow manage to halve the errors which we make in observing each scale. The time required for the atmosphere to wipe out this advantage, and hence the net increase in the time-range of our forecasting, will simply be the doubling-time for the smallest scale. If we could observe all scales down to thunderstorm scale, a further doubling of precision would gain us only a few minutes.

Indeed, we may extrapolate our results to the case where arbitrarily small scales are admitted. We then conclude that the atmosphere possesses an intrinsic range of predictability of perhaps three weeks. At present we are far short of our goal of making the best possible forecasts, and our observation system requires major improvements. However, if the hoped-for improvements are some day realized, still further improvements will not appreciably increase the range of predictability.

Although we feel that the evidence favoring our conclusions is substantial, we must be quick to note that they are based upon a number of assumptions which cannot be rigorously defended. We are a long way from incorporating the true atmospheric equations into our procedure. We are therefore somewhat reluctant to name a maximum range of predictability without including a safety factor.

We must also note that our results apply only to prediction of the weather on a specific date. We say nothing, for example, about the possibility of telling whether next summer will be a warm one or a cool one. What we maintain is that it is not possible to say which days during the coming summer will be the warmer ones or the cooler ones.

The Equations Were Optimistic
A special result of the dynamical-empirical study is that after the errors in those scales which are large enough to appear on weather maps have become noticeable, but before they have become large, further doubling requires somewhat more than two days. This doubling rate is consistent with the one deduced from analogues, but it is appreciably more rapid than that indicated by numerical weather prediction. We must therefore note a particular shortcoming of the latter approach.

In the earlier days of experimentation with the equations of numerical forecasting, it was found that the solutions, after behaving in a reasonable fashion for perhaps several weeks, would suddenly go into wild oscillations, not, of course, observed in
nature. Various computational schemes, which by no means duplicate the manner in which the real atmosphere is prevented from blowing up, were eventually devised to overcome this difficulty. It seems likely that these schemes, which prevent certain computational errors from becoming unduly large, may also have a damping effect upon consequences of errors in the data on which the computations are based, and thereby raise the computed doubling time for these above its proper value. We have tested one such stabilizing scheme, devised by A. Arakawa, which Mintz incorporated into his equations, for this effect.

In short we have repeated the dynamical-empirical study, using as coefficients not those derived from the actual atmospheric laws, but the coefficients which would be appropriate if the Arakawa computation scheme were true of the real atmosphere. We got a five-day doubling time, which is the same as that which Mintz got from his equations by the dynamic method. With the more appropriate coefficients, we obtain a doubling time of 2.5 days.

It thus appears that all three approaches yield nearly the same doubling time for small errors, in scales large enough to be resolved by conventional networks. The process of doubling every two or three days begins not at the initial moment, with the smallest possible errors of observation, but after a day or two, with errors induced by the inevitable errors in the smaller scales. Before the errors become intolerably large the rapid growth should subside.

**Hope for Short-Range Forecasts**

What do these results say as to the possible improvement of weather forecasting? Certainly they offer little hope for those who would extend the two-week goal to a month or more. They are not especially reassuring even for two-week forecasting. In another respect they offer considerable promise.

According to the dynamical-empirical study, if the largest scale of motion not resolved by the observational network has an intrinsic range of predictability of, say, three days, introducing a fine enough network to resolve all scales of motion (an impossible task, of course), would increase the realizability range of predictability of all the larger scales by just three days. Likewise, improving the network so that the largest remaining unresolved scale has an intrinsic range of predictability of one day, instead of three days, would increase the realizable range of predictability of the larger scales by two days. The latter improvements do not seem beyond our capabilities.

Now, to be able to forecast 16 days in advance as well as we could otherwise forecast 14 days in advance would not be a particularly spectacular achievement. But to be able to predict three days ahead as well as we even now predict one day ahead would be a major accomplishment. It is therefore reasonable to anticipate that one outcome of the current efforts to improve the world-wide observational network will be a new level of excellence in short-range forecasting.

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